C.D.S.
Combined Defence Services

EXAM
I
2025

Elementary Mathematics

Time: 2 hour **Total Marks: 100**

Instructions

1. *This Test Booklet contains **100** items (questions). Each item comprises four responses (answers). You will select the response which you want to mark on the Answer Sheet. In case you feel that there is more than one correct response, mark the response which you consider the best. In any case, choose **ONLY ONE** response for each item.*
2. *You have to mark all your responses **ONLY** on the separate Answer Sheet provided. See directions in the Answer Sheet.*
3. ***All** items carry equal marks.*
4. ***Penalty for wrong answers:***
   THERE WILL BE PENALTY FOR WRONG ANSWERS MARKED BY A CANDIDATE IN THE OBJECTIVE TYPE QUESTION PAPERS.
   (i). *There are four alternatives for the answer to every question. For each question for which a wrong answer has been given by the candidate, **one-third** of the marks assigned to that question will be deducted as penalty.*
   (ii). *If a candidate gives more than one answer, it will be treated as a **wrong answer** even if one of the given answers happens to be correct and there will be same penalty as above to that question.*
   (iii). *If a question is left blank, i.e., no answer is given by the candidate, there will be **no penalty** for that question.*

1. A real number M is squared to give the value N. What is the minimum value of $(M + N)$?
(a) −0.25 **(b)** −0.50 **(c)** 0 **(d)** 0.25

2. What is the sum of all 3-digit numbers that give a remainder of 5 when they are divided by 50?
(a) 9005 **(b)** 9540 **(c)** 9600 **(d)** 9640

3. If the average of 64, 69, 72, 75 and x lies between 62 and 76 (excluding 62 and 76), then what is the number of possible integer values of x?
(a) 68 **(b)** 69 **(c)** 70 **(d)** 71

4. Let x, y and z be variables such that $(x + y + z) = k$, where k is a constant. If $(x + z - y) \times (x - z + y)$ is proportional to yz, then $(y + z - x)$ is proportional to:
(a) x **(b)** y **(c)** yz **(d)** xz

5. Let p be the remainder when 7^{84} is divided by 342 and q be the remainder when 7^{84} is divided by 344. What is $(p - q)$ equal to?
(a) $q°$ **(b)** 1 **(c)** 2 **(d)** 6

6. N is the smallest 5-digit number which when divided by 2, 2^2, 2^3, 2^4, ... 2^n leaves a remainder 1. What is the value of n?
(a) 12 **(b)** 13 **(c)** 14 **(d)** 15

7. What is the minimum value of p for which $\frac{1}{532900} + \frac{p^2}{266450} + \frac{p^4}{532900}$ is an integer?
(a) 729 **(b)** 243 **(c)** 27 **(d)** 1

8. If α and β are the roots of the equation $x + a + b = \frac{abx}{ab + ax + bx}$ then what is $(\alpha\beta + \alpha + \beta)$ equal to?
(a) $ab + a + b$ **(b)** $ab - a - b$
(c) $a + b - ab$ **(d)** $-(ab + a + b)$

9. Consider a 2-digit number N. Let P be the product of the digits of the number. If P is added to the square of the digit in the tens place of N, we get 84. If P is added to the square of the digit in the unit place of N, we get 60. What is the value of $P + N$?
(a) 100 **(b)** 110 **(c)** 115 **(d)** 120

10. A mixture of 100 L contains kerosene and turpentine oil in the ratio 3 : 2. What is the minimum quantity of kerosene in litres (whole number) that should be mixed in the mixture so that the resulting mixture has 20% of kerosene?
(a) 10 L **(b)** 20 L
(c) 25 L **(d)** Not possible

11. A lamp is kept on a vertical pole. The height of the top of the lamp above the ground is $\frac{5\sqrt{3}}{2}$ m. The perpendicular distances of the bottom of the pole from two adjacent walls meeting perpendicularly are 0.7 m and 2.4 m. What is the distance of the top of the lamp from the corner point of the walls on the ground?
(a) 3 m **(b)** 5 m
(c) 6 m **(d)** 7 m

12. C is the centre of a circle of radius 20 cm. AB is a chord of length 32 cm. E is a point on AB such that CE = 13 cm. What is AE × EB equal to?
(a) 231 square cm **(b)** 256 square cm
(c) 272 square cm **(d)** 297 square cm

13. The inside of a bowl is part of a sphere. When water is put into the bowl to a depth d, the

water surface becomes a circle of radius 2d. What is the radius of the sphere?

(a) 2.5d **(b)** 2.75d
(c) 3d **(d)** 3.25d

14. In a triangle ABC, AB = 2 cm, BC = 4 cm and AC = 3 cm. The bisector of angle A meets BC at D, the bisector of angle A meets BC and the bisector of angle B meets AD at E. What is AE : ED equal to?

(a) 5 : 4 **(b)** 5 : 3
(c) 4 : 3 **(d)** 3 : 2

15. In a triangle ABC, the bisector of angle A cuts BC at D. If AB + AC = 10 cm and BD : DC = 3 : 1, then what is the length of AC?

(a) 2.5 cm **(b)** 6 cm
(c) 7.5 cm **(d)** 8 cm

16. In a triangle ABC, AB + BC = 7.1 cm, BC + CA = 12.1 cm and CA + AB = 7.2 cm. What is the area of the triangle?

(a) 3 square cm **(b)** 32 square cm
(c) 33 square cm **(d)** 3.3 square cm

17. The adjacent sides of a parallelogram are 10 cm and 8 cm and the angle between them is 150°. What is the area of the parallelogram?

(a) $40\sqrt{3}$ square cm **(b)** 40 square cm
(c) $20\sqrt{3}$ square cm **(d)** 20 square cm

18. The measure of an angle formed by the bisectors of the angles *A* and *C* of the triangle *ABC* is 130°. What is the measure of the angle B?

(a) 65° **(b)** 75° **(c)** 80° **(d)** 85°

19. What is $\log_{10} 2000 + \log_{10} 400 + 4 \log_{10} 25 + 5\log_{10} 20$ equal to?

(a) 10 **(b)** 16 **(c)** 18 **(d)** 20

20. If $\frac{\log_{10}(100001 - 4^x)}{5 - x} = 1$ then what is x equal to?

(a) 0 **(b)** 1 **(c)** 10 **(d)** 100

21. If $2\sin^4\alpha + 2\cos^4\alpha - 1 = 0$, where $0 \le \alpha < \frac{p}{2}$ then what is $\sin 2\alpha + \cos 2\alpha$ equal to?

(a) 0 **(b)** 1
(c) $\frac{\sqrt{3}+1}{2}$ **(d)** $\frac{\sqrt{3}-1}{2}$

22. Consider the following :

I. $1 - \sin^6\alpha = \cos^2\alpha (\cos^4\alpha - 3\cos^2\alpha + 3)$

II. $\cos^8\alpha - \sin^8\alpha = 2\sin^2\alpha (1 - \cos^4\alpha + \sin^2\alpha \cos^2\alpha)$

Which of the above is/are identities?

(a) I only (b) II only
(c) Both I and II (d) Neither I nor II

23. If $p = \frac{1}{\text{cosec}\,\theta + \cot\theta}$ and q = cosec θ, then what is $p^2 - 2pq$ equal to?

(a) −1 **(b)** 0
(c) 1 **(d)** 2

24. Consider the following statements:

I. (cosec α − sec α) is always positive in the first quadrant.

II. (tan α − cot α) is always negative in the first quadrant.

Which of the statements given above is/are correct?

(a) I only **(b)** II only
(c) Both I and II **(d)** Neither I nor II

25. A tower subtends an angle of 60° at a point *A* on the same level as the foot of the tower. *B* is a point vertically above *A* and *AB* = *h*. The angle of depression of the foot of the tower, measured from *B* is 30°. What is the height of the tower?

(a) 2*h* **(b)** 2.5*h*
(c) 3*h* **(d)** 3.5*h*

26. What is $\frac{\sin\theta}{1-\cot\theta} + \frac{\cos\theta}{1-\tan\theta}\left(\theta \ne \frac{\pi}{4}\right)$ equal to?

(a) sin θ + cos θ **(b)** sinθ − cosθ
(c) cos θ − sin θ **(d)** −(sin θ + cos θ)

27. The length of an arc of a circle of radius 4 cm is m cm. What is the magnitude of the angle subtended by the arc at the centre?

(a) m **(b)** $\frac{p}{2}$
(c) $\frac{p}{3}$ **(d)** $\frac{p}{4}$

28. If $\cot^2\theta - 3\sqrt{3}\cot\theta + 6 = 0$, where $\frac{\pi}{6} \le \theta < \frac{\pi}{2}$, then what is the value of $\sin\theta + \cos^2\theta$?

(a) 0 **(b)** 1 **(c)** $\sqrt{3}$ **(d)** $1 + \sqrt{2}$

29. Which of the following equations is/are possible?

I. $\sin^2\theta = \frac{(x+y)^2}{4xy}$ where x and y are positive unequal real quantities.

II. $\sin\theta + \cos\theta = x + \frac{1}{x}$ where x is a positive real quantity.

Select the correct answer using the code given below:

(a) I only **(b)** II only
(c) Both I and II **(d)** Neither I nor II

30. If $m^2 (\sin\theta - 1) + n^2 (\sin\theta + 1) = 0$, where $0 < 0 < \frac{\pi}{2}$ then what is $(m^2 + n^2)\cos\theta - (m^2 - n^2)\cot\theta$ equal to?
(a) $4mn$ **(b)** $2mn$
(c) 1 **(d)** 0

31. If $\sin\alpha + \cos\alpha = \sqrt{2}$, where $0 < \alpha < \frac{p}{2}$, then what is $\sin^3\alpha - \cos^3\alpha$ equal to?
(a) 1 **(b)** $\frac{1}{2}$
(c) $\frac{1}{4}$ **(d)** 0

32. What is $(1 + \cot\alpha - \operatorname{cosec}\alpha)(1 + \tan\alpha + \sec\alpha)$ equal to?
(a) $\frac{1}{2}$ **(b)** 1
(c) 2 **(d)** 4

33. If $\tan\theta = \frac{\sin\alpha - \cos\alpha}{\sin\alpha + \cos\alpha}$, where θ and $\alpha \left(\alpha \neq \frac{\pi}{4}\right)$ are acute angles, then what is $\sqrt{2\sin\theta}$ equal to?
(a) $\sin\alpha - \cos\alpha$ **(b)** $\sin\alpha + \cos\alpha$
(c) $\cos\alpha - \sin\alpha$ **(d)** $\pm(\sin\alpha - \cos\alpha)$

34. For how many values of a does the expression $(\sin\alpha + 2)(\sin\alpha + 4)(\sin\alpha - 2)(\sin\alpha - 4)$ become zero?
(a) No value
(b) One
(c) Two
(d) Four

35. What is the value of x, where $0 \le x < 30°$, satisfying $\tan 3x \tan 6x = 1$?
(a) 0° **(b)** 10° **(c)** 12° **(d)** 15°

36. What is $\frac{(a-b)^2}{(b-c)(c-a)} + \frac{(b-c)^2}{(c-a)(a-b)} + \frac{(c-a)^2}{(a-b)(b-c)} - 3$ equal to, where $a \neq b \neq c$?
(a) 0 **(b)** 3
(c) $a + b + c$ **(d)** $3(a-b)(b-c)(c-a)$

37. Given that $\frac{100 \times 99 \times 98 \times \ldots \times 3 \times 2 \times 1}{100^n}$ is an integer. What is the largest value of n for which this is true?
(a) 20 **(b)** 21
(c) 24 **(d)** None of the above

38. A man starting from a place P went x metre ($x > 120$ m) East before turning South. He went 40 m straight before turning to the West. He went 60 m to reach a place Q. From Q he went 200 m North and reached a place R. If PR = 200 m, then what is x equal to?
(a) 150 m **(b)** 180 m
(c) 200 m **(d)** 240 m

39. If $x^2 + y^2 + z^2 = 3$, where x, y and z are integers, then how many values can $(xy + yz + zx)$ have?
(a) One **(b)** Two
(c) Three **(d)** Four

40. If x, y and z are real numbers such that $x + y + z = 10$ and $xy + yz + zx = 18$, then what is the value of $x^3 + y^3 + z^3 - 3xyz$?
(a) 400 **(b)** 440
(c) 460 **(d)** 500

41. What is $\sqrt{17 - 4\sqrt{15}} + \sqrt{8 - 2\sqrt{15}}$ equal to?
(a) $\sqrt{3}$ **(b)** $2\sqrt{3}$
(c) $2(\sqrt{5} - \sqrt{3})$ **(d)** $2(\sqrt{5} + \sqrt{3})$

42. What is the maximum value of the sum of the numbers 36, 33, 30, 27, 24, ... ?
(a) 240 **(b)** 237 **(c)** 234 **(d)** 231

43. There are two natural numbers m and n ($m > n$). When m is divided by 12, it leaves a remainder 4. When n is divided by 12, it leaves a remainder 6. Which of the following statements is/are correct?
I. The remainder when $(m + n)$ is divided by 12 is 10.
II. The remainder when $(m - n)$ is divided by 12 is 10.
Select the correct answer using the code giver below:
(a) I only **(b)** II only
(c) Both I and II **(d)** Neither I nor II

44. If $(x + y) : (y + z) : (z + x) = 3 : 5 : 6$ and $x + y + z = 14$, then what is $x^2 + y^2 + z^2$ equal to?
(a) 81 **(b)** 84
(c) 87 **(d)** 90

45. The ratio of the sum of two numbers to their difference is 5 : 1. What is the ratio of the sum of their squares to the difference of their squares?
(a) 13 : 5 **(b)** 25 : 1
(c) 9 : 4 **(d)** 16 : 1

46. Travelling at $3/5^{th}$ of his usual speed, a man is late by 20 minutes. What is the usual time if he travels at his usual speed?
(a) 25 minutes **(b)** 30 minutes
(c) 32 minutes **(d)** 35 minutes

47. What is the remainder when $2^p - 1$ is divided by p, where $p > 5$ is a prime number?
(a) 1 **(b)** 2 **(c)** 3 **(d)** 4

48. What is the number of factors of $24^3 - 16^3 - 8^3$?
(a) 33 **(b)** 30 **(c)** 28 **(d)** 24

49. What is the least number of complete years in which a sum of money put out at 20% compound interest (compound annually) will be more than doubled?
(a) 2 **(b)** 3 **(c)** 4 **(d)** 5

50. A train of certain length takes time t to pass completely through a station of length x. The same train with the same speed takes time $2t$ to pass completely through another station of length y. What is the time taken by the train to pass completely through a station of length $(x + y)$?

(a) $\frac{(2yt + xt)}{(y - x)}$ **(b)** $\frac{(yt + xt)}{(y - x)}$

(c) $\frac{(3yt - xt)}{(2y - x)}$ **(d)** $\frac{(2yt - xt)}{(y - x)}$

Consider the following for the next two (02) items that follow:

A frustum of a right cone has a top of diameter $2k$, bottom of diameter $2.5k$ and height k.

51. What is the whole surface area of the frustum?

(a) $\frac{39\pi k^2}{8}$ **(b)** $\frac{41\pi k^2}{8}$

(c) $\frac{43\pi k^2}{8}$ **(d)** $\frac{45\pi k^2}{8}$

52. What is the volume of the frustum?

(a) $\frac{61\pi k^3}{48}$ **(b)** $\frac{59\pi k^3}{48}$

(c) $\frac{57\pi k^3}{48}$ **(d)** $\frac{53\pi k^3}{48}$

Consider the following for the next two (02) items that follow:

ABC is a triangle right-angled at B. The perimeter of the triangle is 24 cm, and the different between the sum of the perpendicular sides and the hypotenuse is 4 cm.

53. What is the area of the triangle ABC?

(a) 18 square cm **(b)** 24 square cm

(c) 36 square cm **(d)** 48 square cm

54. A circle is inscribed in the triangle. What is its radius?

(a) 1 cm **(b)** 1.5 cm **(c)** 2 cm **(d)** 2.5 cm

Consider the following for the next two (02) items that follow:

A circle M of radius 8 cm touches externally another circle N of radius 16 cm. Let P and Q be the points where the common tangent touches the circles M and N, respectively.

55. What is the length of the common tangent PQ?

(a) 16 cm **(b)** $16\sqrt{2}$ cm

(c) 24 cm **(d)** $24\sqrt{2}$ cm

56. If U and V are the centres of the circles M and N respectively, then what is the area of the quadrilateral formed by the points P, Q, V and U?

(a) $192\sqrt{2}$ square cm **(b)** 192 square cm

(c) $96\sqrt{2}$ square cm **(d)** 96 square cm

Consider the following for the next two (02) items that follow:

The perimeter of a triangle ABC is 105 cm. The altitudes AD, BE and CF are in the ratio 3 : 5 : 6.

57. What is AB : BC : CA equal to?

(a) 10 : 6 : 5 **(b)** 5 : 10 : 6

(c) 6 : 5 : 3 **(d)** 3 : 5 : 6

58. What is the approximate area of the triangle ABC?

(a) 175 square cm **(b)** 190 square cm

(c) 205 square cm **(d)** 285 square cm

Consider the following for the next two (02) items that follow:

A pot is made from a hollow sphere of inner radius 20 cm by cutting its upper portion horizontally. The height of the pot is 30 cm.

59. What is the inner radius of the circular opening of the pot so formed?

(a) $10\sqrt{2}$ cm **(b)** 15 cm

(c) $10\sqrt{3}$ cm **(d)** 12 cm

60. What is the angle made by the line joining the centre of the sphere and any point on the rim of the circular opening with a vertical line passing through the centre?

(a) $\frac{\pi}{3}$ **(b)** $\frac{\pi}{4}$ **(c)** $\frac{\pi}{6}$ **(d)** $\frac{\pi}{12}$

Consider the following for the next two (02) items that follow:

A hall is of length l, breadth b and height h. The maximum distance between any two points (say P and Q) inside the hall is 14 m, whereas the maximum distance between any two points (say P and R) on the floor is $6\sqrt{5}$ m.

61. What is h equal to?

(a) 3.5 m **(b)** 4 m **(c)** 4.5 m **(d)** 5 m

62. If α is the angle between the line joining P and Q, and the line joining P and R, then what is $\cos\alpha$ equal to?

(a) $\frac{2\sqrt{5}}{7}$ **(b)** $\frac{3\sqrt{5}}{7}$ **(c)** $\frac{1}{3}$ **(d)** $\frac{2}{3}$

Consider the following for the next two (02) items that follow:

The sides of an open box are 0.5 cm thick and bottom is 1 cm thick. The internal length, breadth and depth are, respectively, 14 cm, 10 cm and 8 cm. It is completely filled with water.

63. If the material weight 2000 kg per cubic metre, then what is the weight of the material used in the construction of the box?

(a) 360 gm **(b)** 365 gm
(c) 720 gm **(d)** 730 gm

64. If water weights 1000 kg per cubic metre, then what is the weight of the box with water?
(a) 1.850 kg **(b)** 1.900 kg
(c) 2.050 kg **(d)** 2.100 kg

Consider the following for the next two (02) items that follow:

ABC is a triangle right-angled at A. Further AB = 8 cm and BC = 10 cm. D is the point on BC such that AD is perpendicular to BC.

65. What is AD equal to?
(a) 4.8 cm **(b)** 5.0 cm
(c) 5.2 cm **(d)** 5.4 cm

66. What is the ratio of area of triangle ADC to area of triangle ADB?
(a) 7 : 15 **(b)** 9 : 16
(c) 2 : 3 **(d)** 3 : 4

Consider the following for the next two (02) items that follow:

The annual rainfall at a place is 40 cm. The weight of water is 1 metric tonne per cubic metre.

67. What is the volume of rainfall in cubic metre per hectare?
(a) 40 **(b)** 400 **(c)** 4000 **(d)** 40000

68. What is weight of water (in metric tonnes) of annual rainfall falling there on a hectare of land?
(a) 40 **(b)** 400 **(c)** 4000 **(d)** 40000

Consider the following for the next two (02) items that follow:

The angle at the vertex of a conical body is 120°

69. What is the ratio of the radius of the conical body to its slant height?
(a) 1 : 2 **(b)** $\sqrt{3}:1$ **(c)** $\sqrt{3}:2$ **(d)** $\sqrt{2}$ 1

70. If the sum of slant height, height and radius is $(9+3\sqrt{3})$ cm, then what is the volume of the cone?
(a) 27 m cubic cm **(b)** $18\sqrt{3}$ m cubic cm
(c) 24 m cubic cm **(d)** $27\sqrt{3}$ m cubic cm

71. A person sells article X for ` 34,500 and makes a profit of 15%. He sells article Y at a loss of 10%. He neither loses nor gains on the whole because of these two transactions. What is the selling price of article Y?
(a) ` 40000 **(b)** ` 40500
(c) ` 41000 **(d)** ` 51500

72. 100 quintals is what per cent of 10 metric tonnes?
(a) 1 % **(b)** 10 %
(c) 100 % **(d)** 1000 %

73. A circle is inscribed in an equilateral triangle. The radius of the circle is 2 cm. What is the area of the triangle?
(a) $12\sqrt{3}$ square cm **(b)** 12 square cm
(c) $9\sqrt{3}$ square cm **(d)** 9 square cm

74. The sides of a triangle are k, 1.5 k and 2.25 k. What is the sum of the squares of its medians?
(a) $359k^2/64$ **(b)** $379k^2/64$
(c) $389k^2/64$ **(d)** $399k^2/64$

75. If $2s = a + b + c$, then what is $s(s-a)(s-b)(s-c)\left[\frac{1}{s-a}+\frac{1}{s-b}+\frac{1}{s-c}-\frac{1}{s}\right]$ equal to?
(a) abc **(b)** $2abc$
(c) $4abc$ **(d)** $ab + bc + ca$

76. How much will ` 10000 amount to in 1 year's time at a 4% rate of interest per annum if the interest is compounded once in every three months? (take approximately)
(a) ` 10,406 **(b)** ` 10,416
(c) ` 10,426 **(d)** ` 10,436

77. If $p = 0.09$, then what is the value of $70p^2 + 43p - 5$?
(a) – 1 **(b)** 0
(c) 1 **(d)** 10

78. What is the remainder when 2^{101} is divided by 101?
(a) 1 **(b)** 2
(c) 5 **(d)** 7

79. If p(≠ 0) and q(≠ 0) are the roots of the equation $x^2 + px + q = 0$, then what is $p^2 + q^2$ equal to?
(a) 2 **(b)** 3 **(c)** 4 **(d)** 5

80. The equations $x^2 + px + q = 0$ and $x^2 + qx + p = 0$ p(≠ q) have a common root. What is the value of $(p + q)$?
(a) – 1 **(b)** 0
(c) 1 **(d)** 2

81. If $x^2 - 5x + 4$ is a factor of $x^4 - px^2 + q$. then what are the values of p and q, respectively?
(a) 17, 16 **(b)** 16, 17
(c) 15, 16 **(d)** 16, 15

82. If two quadratic equations $px^2 + px + 4 = 0$ and $x^2 + qx + q = 0$ have a common root 2, then what is $p + q$ equal to?
(a) – 3 **(b)** – 2 **(c)** 0 **(d)** 3

83. Which is the HCF of the polynomials $x^8 + x^4 + 1$ and $x^4 + x^2 + 1$?
(a) 1 **(b)** $x^4 - x^2 + 1$
(c) $x^4 + x^2 + 1$ **(d)** $x^4 - x^2 - 1$

84. An arc AB of a circle subtends an angle x radian at the centre O. If the area of the sector AOB is equal to half of the square of length of arc AB, then what is x equal to?
(a) $\frac{1}{4}$ **(b)** $\frac{1}{2}$ **(c)** 1 **(d)** 2

85. Consider the following statements in respect of prime numbers p and q:

I. Their LCM is always an odd number.
II. Sum of their LCM and HCF is always an even number.

Which of the statements given above is/are correct?

(a) I only **(b)** II only
(c) Both I and II **(d)** Neither I nor II

Consider the following for the next five (05) items that follow:

The frequency distribution of 205 observations on X is given below?

X	3	5	6	7
Frequency	f	$f+2$	$f-3$	$f+6$

86. What is the value of f?
(a) 50 **(b)** 60
(c) 70 **(d)** 80

87. What is the median of the frequency distribution?
(a) 3
(b) 5
(c) 6
(d) It cannot be determined from the given data.

88. What is the mode of the frequency distribution?
(a) 5
(b) 6
(c) 7
(d) It cannot be determined from the given data

89. What is the most appropriate graphical representation of the given frequency distribution of X?
(a) Bar diagram
(b) Histogram
(c) Frequency polygon
(d) Pie chart

90. What is the mean of the frequency distribution?
(a) 3.29 **(b)** 4.29
(c) 5.29 **(d)** 6.29

Consider the following for the next two (02) items that follow:

The frequency distribution of marks obtained by students in an English examination is given below:

Marks obtained	**Number of Students**
Below 40	50
Below 50	125
Below 60	210
Below 70	315
Below 80	350

91. What is the number of students who scored between 60 and 70 marks?
(a) 105 **(b)** 110
(c) 205 **(d)** 210

92. What is the number of students who scored more than 50 marks?
(a) 100 **(b)** 125 **(c)** 200 **(d)** 225

Consider the following for the next three (03) items that follow:

Circulation figures (in thousands) of different newspapers (A, B, C, D and E) for five years are given below:

Year	A	B	C	D	E
2019	20	10	15	8	20
2020	12	12	18	12	12
2021	24	14	17	14	15
2022	26	10	16	15	9
2023	22	16	14	16	11

93. In which of the given years was the circulation of the newspaper D close to its average circulation over all the years?
(a) 2020 and 2021 **(b)** 2022 and 2023
(c) 2022 only **(d)** 2020 only

94. In which of the years from 2019 to 2022 was the circulation of the newspaper D close to the average circulation of all the newspapers in that year?
(a) 2019 **(b)** 2020
(c) 2021 **(d)** 2022

95. How many cases are there in which average of the circulation for an individual newspaper was more than the average of the circulation of all the newspapers?
(a) One **(b)** Two
(c) Three **(d)** Four

Consider the following for the next three (03) items that follow:

Study the following Pie charts:

Chart-I: Number of houses : 1,20,000

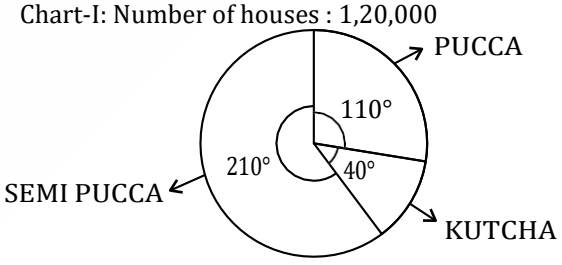


Chart-II: Population : 21,00,000

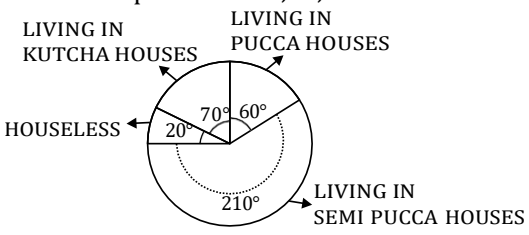
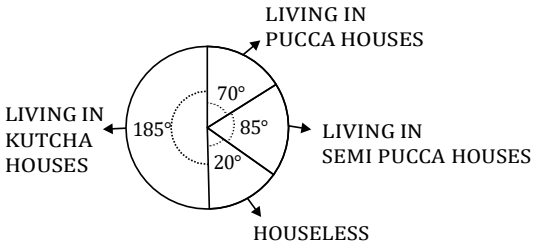


Chart-III : Number of families : 3,60,000



96. On an average, how many persons live in every pucca house?

(a) 8 **(b)** 8.54

(c) 9.54 **(d)** 7.54

97. If 5000 more kutcha houses are built, then what will be the approximate change in angle for kutcha houses in Pie chart-I?

(a) 11° **(b)** 13°

(c) 15° **(d)** 23°

98. If 300 families from the "Houseless" category shift into kutcha houses, what will be the average number of families in every Kutcha house?

(a) 15 **(b)** 13.9 **(c)** 12.9 **(d)** 10.5

99. Marks obtained by 60 students in a test (Maximum Marks-100) are given below:

60, 82, 63, 40, 88, 80, 24, 60, 09, 58, 44, 74
57, 60, 63, 30, 31, 64, 00, 62, 37, 54, 10, 99
35, 83, 58, 33, 92, 43, 66, 41, 78, 72, 32, 33
65, 11, 60, 17, 53, 93, 57, 03, 29, 31, 82, 50
63, 15, 41, 55, 72, 42, 32, 23, 30, 48, 67, 34

What is the percentage (approximate) of students getting marks higher than or equal to 60 but less than 80 ?

(a) 25.0 **(b)** 26.7 **(c)** 28.3 **(d)** 30.3

100. The arithmetic mean of 100 observations was 50. Later on, it was found that two observations were misread as 82 and 6 instead of 182 and 56. What is the value of the corrected mean?

(a) 50.1 **(b)** 51.5

(c) 52.3 **(d)** 53.4

OSWAAL 360

Answer Key

Q. No	Answer Key	Topic's Name	Chapter's Name
1	a	Maxima and minima	Quadratic equation
2	b	Sum of AP	Sequence and series
3	b	General term of AP	Sequence and series
4	c	Ratio and proportion	Ratio and proportion
5	a	Remainder	Binomial theorem
6	c	Divisibility	Number system
7	c	Minimum value	Number system
8	b	Roots of equation	Quadratic equation
9	b	Number system	Number system
10	d	Percentage	Ratio and proportion
11	b	Distance between points	Coordinate geometry
12	a	Chord of circle	Circle
13	a	Sphere	Sphere
14	a	Angle bisector	Triangle
15	a	Angle bisector	Triangle
16	d	Area of triangle	Triangle
17	b	Area of parallelogram	Parallelogram
18	c	Angle bisector	Triangle
19	c	Log	Algebra
20	a	Log	Algebra
21	b	Trigonometric identities	Trigonometric identities
22	a	Trigonometric identities	Trigonometric identities
23	a	Trigonometric identities	Trigonometric identities
24	d	Trigonometric identities	Trigonometric identities
25	c	Height and distance	Height and distance
26	a	Trigonometric identities	Trigonometric identities
27	d	Arc of circle	Circle
28	b	Solution of equation	Trigonometric equation
29	d	Solution of equation	Trigonometric equation
30	d	Solution of equation	Trigonometric equation
31	d	Solution of equation	Trigonometric equation
32	c	Trigonometric identities	Trigonometric identities
33	a	Trigonometric identities	Trigonometric identities
34	a	Solution of equation	Trigonometric equation
35	b	Trigonometric identities	Trigonometric identities
36	a	Identities	Quadratic equation
37	d	Exponent	Permutation and combination
38	b	Pythagoras theorem	Coordinate geometry
39	b	Solution of equation	Number system
40	c	Identities	Number system
41	a	Surds	Surds and indices
42	c	Sum of AP	Sequence and series
43	c	Remainder	Number system
44	b	Real number	Number system
45	a	Ratio and proportion	Ratio and proportion
46	b	Speed and time	Speed and time
47	a	Remainder	Number system
48	a	Factors	Permutation and combination
49	c	Compound interest	Simple and compound interest
50	d	Speed and time	Speed and time

51	**a**	Frustum of cone	Mensuration
52	**a**	Frustum of cone	Mensuration
53	**b**	Area of triangle	Solution of triangle
54	**c**	Inradius	Solution of triangle
55	**b**	Common tangent	Circle
56	**a**	Common tangent	Circle
57	**b**	Median	Triangle
58	**d**	Area of triangle	Solution of triangle
59	**c**	Sphere	Three dimensional
60	**a**	Sphere	Three dimensional
61	**b**	Cuboid	Mensuration
62	**b**	Cuboid	Mensuration
63	**d**	Volume of box	Mensuration
64	**a**	Weight of box	Mensuration
65	**a**	Area of triangle	Triangle
66	**b**	Similar triangle	Triangle
67	**c**	Number system	Number system
68	**c**	Number system	Number system
69	**c**	Cone radius	Mensuration
70	**a**	Volume of cone	Mensuration
71	**b**	Profit and loss	Profit and loss
72	**c**	Number system	Number system
73	**a**	Area of triangle	Solution of triangle
74	**d**	Appollonious theorem	Triangle
75	**a**	Solution of equation	Solution of triangle
76	**a**	Compound interest	Simple and compound interest
77	**b**	Number system	Number system
78	**b**	Divisibility	Number system
79	**d**	Roots of equation	Quadratic equation
80	**a**	Common roots	Quadratic equation
81	**a**	Factors	Polynomial
82	**b**	Common roots	Quadratic equation
83	**c**	HCF	Number system
84	**c**	Area of sector	Circle
85	**d**	LCM	Number system
86	**a**	Frequency	Statistics
87	**c**	Median	Statistics
88	**c**	Mode	Statistics
89	**b**	Graphical representation	Statistics
90	**c**	Mean	Statistics
91	**a**	Sets	Algebra
92	**d**	Sets	Algebra
93	**a**	Average	Number system
94	**d**	Average	Number system
95	**b**	Average	Number system
96	**c**	Pie chart	Statistics
97	**b**	Pie chart	Statistics
98	**b**	Pie chart	Statistics
99	**b**	Percentage	Number system
100	**b**	Mean	Statistics

■■■

CDS
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EXAM I 2025

Solutions Mathematics

ANSWERS WITH EXPLANATIONS

1. Option (a) is correct.

Explanation: Given, $N = M^2$

- $M + N = M + M^2$

$$= M + M^2 + \frac{1}{4} - \frac{1}{4}$$

$$= \left(M + \frac{1}{2}\right)^2 - \frac{1}{4}$$

M + N is minimum if $M + \frac{1}{2} = 0$

- $(M + N)$ minimum $= -\frac{1}{4}$

2. Option (b) is correct.

Explanation: Let a three digit number be N

$N = 50K + 5, K \in I^+$

Least value of K = 2

Maximum value of K = 19

Numbers are, 105, 155,. ..., 955

Sum = 105 + 155 +....., 955

$$\text{Sum of A.P.} = \frac{n}{2}(a + L)$$

n = number of terms

a = first term

L = last term

$$\text{Sum} = \frac{18}{2}(105 + 955)$$

$$= \frac{18 \times 1060}{2}$$

$= 18 \times 530$

$= 9540$

3. Option (b) is correct.

Explanation: Average= $\frac{\text{sum of all numbers}}{n}$

$$= \frac{64 + 69 + 72 + 75 + x}{5}$$

$$= \frac{280 + x}{5}$$

$$= 56 + \frac{x}{5}$$

According to the problem,

$$62 < 56 + \frac{x}{5} < 76$$

$$6 < \frac{x}{5} < 20$$

$30 < x < 100$

x Î {31, 32, ..., 99}

$t_n = a + (n - 1)d$

a = first term of A.P.

d = common difference

n = number of terms

$99 = 31 + (n - 1) \times 1$

$68 = n - 1$

$n = 69$

4. Option (c) is correct.

Explanation: Given, $x + y + z = k$

Let I is a constant

- $(x + z - y)(x - z + y) = Iyz\, x^2 - (y - z)^2 = Iyz$

$\Rightarrow \quad Iyz = x^2 - \{(y - z)^2 + 4yz - 4yz\}$

$\Rightarrow \quad Iyz = x^2 - \{(y + z)^2 - 4yz\}$

$\Rightarrow \quad Iyz = x^2 - (y + z)^2 + 4yz$

$\Rightarrow \quad Iyz - 4yz = (x + y + z)(x - y - z)$

$\Rightarrow \quad yz(4 - I) = k(y + z - x)$

$\Rightarrow \quad y + z - x = \frac{4 - \lambda}{k}(yz)$

$\rightarrow \quad y + z - x \;\mu\; yz$

5. Option (a) is correct.

Explanation: We know that,

$7^3 = 343$

$7^{84} = (7^3)^{28}$

$= (343)^{28}$

$(342 + 1)^{28} = {}^{28}C_0(342)^{28} + ... + {}^{28}C_{27}(342) + {}^{28}C_{28}$

If $(342)^{28}$ is divided by 342 then the remainder is ${}^{28}C_{28} = 1$

Now, $(343)^{28} = (344 - 1)^{28}$

$(344 - 1)^{28} = {}^{28}C_0(344)^{28} - {}^{28}C_1(344)^{27} + ... {}^{28}C_{27}344 + {}^{28}C_{28}$

If $(343)^{28}$ is divided by 344 then the remainder is ${}^{28}C_{28} = 1$

∴ $p - q = 0$

6. Option (c) is correct.

Explanation: If a number is divided by 2^n, then it will be divisible by 2, 2^2,....., 2^{n-1}.

∴ $N = 2^n + 1$

N is a five digit number,

$2^{10} = 1024$

$\Rightarrow 2^{10} \times 2^3 = 1024 \times 8$

$= 8192$

$\Rightarrow 2^{13} \times 2 = 8192 \times 2$

$= 16384$

$\rightarrow N = 2^{14} + 1$

7. Option (c) is correct.

Explanation: Given, $\frac{1}{532900} + \frac{p^2}{266450} + \frac{p^4}{532900}$

is an integer

$\rightarrow \frac{1}{532900} + \frac{2p^2}{532900} + \frac{p^4}{532900}$

$= \frac{(p^2+1)^2}{(730)^2}$

For minimum value of 'p'

$p^2 + 1 = 730$

$\Rightarrow p^2 = 729$

$\Rightarrow p = 27$

Note, the given problem is wrong, a small correction is done to get answer.

8. Option (b) is correct.

Explanation: $x + a + b = \frac{abx}{ab + ax + bx}$ has roots α and β.

∴ $(x + a + b)(ab + x(a + b)) = abx$

$abx + (a + b)ab + x^2(a + b)$

$+ x(a + b)^2 = abx$

$x^2(a + b) + x(a + b)^2 + ab(a + b) = 0$

$x^2 + x(a + b) + ab = 0$

∴ $\alpha\beta + \alpha + \beta = ab - a - b$

9. Option (b) is correct.

Explanation: Let, $N = 10a + b$

$a, b \in \{0, 1, 2, \ldots\ldots\ldots, 9\}$

and $a \neq 0$

$P = a \times b$

Given, $a^2 + ab = 84$.....(1)

$b^2 + ab = 60$.....(2)

Adding both equations,

$a^2 + b^2 + 2ab = 84 + 60$

$(a + b)^2 = 144$

$a + b = 12$

From equation (1) we get

$a \times 12 = 84$

$a = 7,$

$b = 5$

$P = 7 \times 5 = 35$

$N = 75$

$P + N = 75 + 35 = 110$

10. Option (d) is correct.

Explanation: Kerosene, Turpentine = 3 : 2

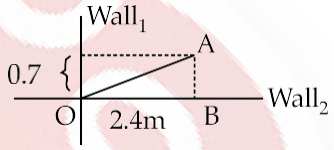
∴ Kerosene in mixture = $100 \times \frac{3}{5} = 60$ L

Turpentine = 40 L

Currently, 60% kerosene is present, by adding kerosene its percentage will increase, hence not possible

11. Option (b) is correct.

Explanation:



The pole is present at point 'A'

$OA = \sqrt{(2.4)^2 + (0.7)^2}$

$= 2.5$

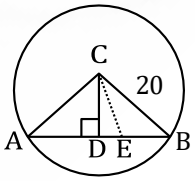
∴ Required distance = $\sqrt{(2.5)^2 + \left(\frac{5\sqrt{3}}{2}\right)^2}$

$= 6.25 + 18.75$

$= 5$ m

12. Option (a) is correct.

Explanation:



Let the midpoint of AB be is D.

$BD = AD = 16$

$BD^2 + CD^2 = BC^2$

$16^2 + CD^2 = 20^2$

$CD^2 = 400 - 256$

$= 144$

∴ $CD = 12$

Given, $CE = 13$

$DE^2 + CD^2 = CE^2$

$DE^2 + 12^2 = 13^2$

$DE^2 = 169 - 144$

∴ $DE = 5$

$AE = 16 + 5 = 21$

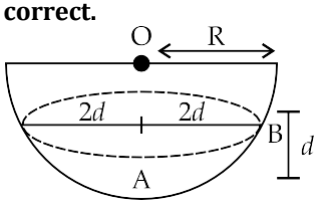
$BE = 16 - 5 = 11$

$AE \times BE = 21 \times 11$

$= 231$ cm^2

13. Option (a) is correct.

Explanation:



Let the radius of sphere = R

$OA = R - d$

$\Rightarrow \quad OB = R$

$\Rightarrow \quad AB = 2d$

Applying Pythagoras Theorem,

$\Rightarrow \quad OA^2 + AB^2 = OB^2$

$\Rightarrow \quad (R - d)^2 + 4d^2 = R^2$

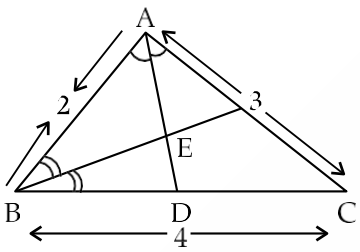
$\Rightarrow \quad R^2 + d^2 - 2Rd + 4d^2 = R^2$

$\Rightarrow \quad 5d^2 - 2Rd = 0$

$\therefore \quad R = \frac{5d}{2}$

14. Option (a) is correct.

Explanation:



Apply Angle Bisector Theorem in

$\Delta ABC, \quad \frac{BD}{DC} = \frac{AB}{AC}$

$\frac{BD}{DC} = \frac{2}{3}$

$\therefore \quad BD = \frac{4 \times 2}{2+3} = \frac{8}{5}$

Now, ΔABD,

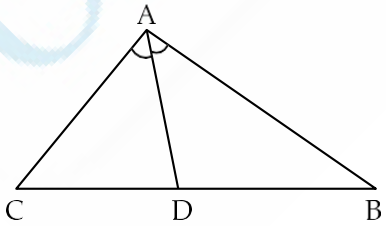
$\frac{AE}{ED} = \frac{AB}{BD}$

$= \frac{2}{8/5}$

$\therefore \quad \frac{AE}{ED} = \frac{5}{4}$

15. Option (a) is correct.

Explanation:



$AB + AC = 10$

$BD : DC = \frac{3}{1}$

$\frac{AB}{AC} = \frac{BD}{DC} = \frac{3}{1}$

$\therefore \quad AB = 10 \times \frac{3}{4}$

$AB = \frac{15}{2}$

$AC = \frac{5}{2}$

16. Option (d) is correct.

Explanation:

$AB + BC = 7.1 \quad ...(i)$

$BC + CA = 12.1 \quad ...(ii)$

$CA + AB = 7.2 \quad ...(iii)$

Adding all three equations,

$2(AB + BC + CA) = 26.4$

$AB + BC + CA = 13.2 \quad ...(iv)$

Now from equations (i), (ii), (iii) and (iv) we get, $CA = 6.1$

$AB = 1.1$

$BC = 6.0$

$AB^2 + BC^2 = 6^2 + 1.1^2$

$= 37.21$

showing as a subscript, but all letters should be in small letter superscript

$= CA^2$

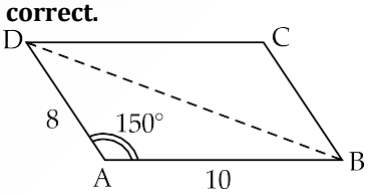
$\therefore$ ABC is a right-angled triangle.

$\text{Area} = \frac{1}{2} \times 6 \times 1.1$

$= 3.3 \text{ cm}^2$

17. Option (b) is correct.

Explanation:



Area of $\Delta ABD = \frac{1}{2} \times AB \times AD \sin(150°)$

$= \frac{1}{2} \times 8 \times 10 \times \frac{1}{2}$

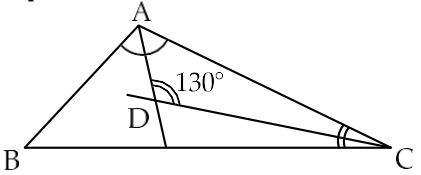
$= 20$

$\therefore \quad$ Total area $= 20 \times 2$

$= 40 \text{ cm}^2$

18. Option (c) is correct.

Explanation:



$$\angle DAC = \frac{A}{2}$$

$$\angle DCA = \frac{C}{2}$$

$$\therefore \frac{A}{2} + \frac{C}{2} + 130° = 180°$$

$$\frac{A+C}{2} = 50°$$

$$A + C = 100°$$

$$\angle B = 180° - 100°$$

$$= 80°$$

19. Option (c) is correct.

Explanation: Given,

$S = \log_{10} 2000 + \log_{10} 400 + 4\log_{10} 25 + 5\log_{10} 20$

We know that, $\log a + \log b = \log ab$

$n\log a = \log(a^n)$

$\therefore S = \log_{10}(2000 \times 400) + \log_{10}(25)^4 + \log_{10}(20)^5$

$= \log_{10}(8 \times 10^5 \times (5)^8 \times 10^5 \times 2^5)$

$= \log_{10}(10^{10} \times (5 \times 2)^8)$

$= \log_{10} 10^{18}$

$= 18 \log_{10} 10$

$= 18$

20. Option (a) is correct.

Explanation: Given, $\frac{\log_{10}(100001 - 4^x)}{5 - x} = 1$

$\text{Log}_{10}(100001 - 4^x) = 5 - x$

(If $\log_d b = k$ then $b = a^k$)

$\therefore 100001 - 4^x = 10^{5-x}$

$10^5 + 1 = 10^5.10^{-x} + 4^x$

$10^5 + 1 = \frac{10^5}{10^x} + 4^x$

$10^5.10^x + 10^x = 10^5 + 10^x.4^x$

If $x = 0$ then both sides are equal.

21. Option (b) is correct.

Explanation: Given, $2\sin^4\alpha + 2\cos^4\alpha - 1 = 0$...(i)

We know that, $\sin^2\alpha + \cos^2\alpha = 1$

Squaring on both side,

$\sin^4\alpha + \cos^4\alpha + 2\sin^2\alpha.\cos^2\alpha = 1$ ...(ii)

From equations (i) and (ii),

$2(1 - 2\sin^2\alpha.\cos^2\alpha) - 1 = 0$

$1 - 4\sin^2\alpha.\cos2\alpha = 0$

$1 - (2\sin\alpha.\cos\alpha)^2 = 0$

$(\sin2\alpha)^2 = 1$

$(\because \sin2\alpha = 2\sin\alpha\cos\alpha)$

$\sin2\alpha = 1 \quad (\because 0 < 2\alpha < \pi)$

$\cos2\alpha = 0$

$\therefore \sin2\alpha + \cos2\alpha = 1$

22. Option (a) is correct.

Explanation: I. Let $P = 1 - \sin^6\alpha$

$\{\because a^3 - b^3 = (a - b)(a^2 + b^2 + ab)\}$

$P = 1^3 - (\sin^2\alpha)^3$

$= (1 - \sin^2\alpha)(1 + \sin^4\alpha + \sin^2\alpha)$

$P = \cos^2\alpha(1 + \sin^2\alpha(1 + \sin^2\alpha))$

$\{\because \sin^2\alpha = 1 - \cos^2\alpha\}$ P

$= \cos^2\alpha(1 + (1 - \cos^2\alpha)(2 - \cos^2\alpha))$

$= \cos^2\alpha(1 + 2 - 3\cos^2\alpha + \cos^4\alpha)$

$= \cos^2\alpha(\cos^4\alpha - 3\cos^2\alpha + 3)$

$\therefore$ I is identity

Now, II Let $Q = \cos^8\alpha - \sin^8\alpha$

$Q = (\cos^4\alpha) - (\sin^4\alpha)^2$

$= (\cos^4\alpha + \sin^4\alpha)(\cos^4\alpha - \sin^4\alpha)$

$= (\cos^4\alpha + \sin^2\alpha(1 - \cos^2\alpha))$

$(\cos^4\alpha - \sin^4\alpha)$

$Q = \{\cos^4\alpha + \sin^2\alpha - \sin^2\alpha\cos^2\alpha\}$

$\{\cos^2\alpha + \sin^2\alpha\}\{\cos^2\alpha - \sin^2\alpha\}$

$= \{1 - 2\sin^2\alpha\}\{\cos^4\alpha + \sin^2\alpha - \sin^2\alpha\cos^2\alpha\}$

$\therefore$ II is not an identity

23. Option (a) is correct.

Explanation: Given, $p = \frac{1}{\text{cosec}\theta + \cot\theta}$

$q = \text{cosec}\,\theta$

We know that,

$\text{cosec}^2\theta - \cot^2\theta = 1$

$\therefore (\text{cosec}\,\theta + \cot\theta)(\text{cosec}\,\theta - \cot\theta) = 1$

$\rightarrow p = \text{cosec}\,\theta - \cot\theta$

$p^2 - 2pq = \text{cosec}^2\theta + \cot^2\theta - 2\text{cosec}\,\theta\cot\theta - 2(\text{cosec}\,\theta - \cot\theta)\text{cosec}\,\theta$

$= \text{cosec}^2\theta + \cot^2\theta - 2\,\text{cosec}\,\theta\cot\theta - 2\,\text{cosec}^2\theta + 2\,\text{cosec}\,\theta\cot\theta$

$= -(\text{cosec}^2\theta - \cot^2\theta)$

$= -1$

24. Option (d) is correct.

Explanation: I, let $p = \text{cosec}\,\alpha - \sec\alpha$

$= \frac{1}{\sin\alpha} - \frac{1}{\cos\alpha}$

$= \frac{\cos\alpha - \sin\alpha}{\cos\alpha\sin\alpha}$

$\cos\alpha - \sin\alpha > 0$ if $0 < \alpha < \frac{\pi}{4}$

$\cos\alpha - \sin\alpha < 0$ if $\frac{\pi}{4} < a < \pi$

$\therefore$ I is wrong

II : Let $Q = \tan\alpha - \cot\alpha$

$Q = \frac{\sin\alpha}{\cos\alpha} - \frac{\cos\alpha}{\sin\alpha}$

$$= \frac{\sin^2\alpha - \cos^2\alpha}{\cos\alpha \sin\alpha}$$

$$= -\left(\frac{2\cos 2\alpha}{\sin 2\alpha}\right)$$

$$Q = -2\cot 2\alpha$$

$\{\because \cos 2\alpha = \cos^2\alpha - \sin^2\alpha\}$

$\sin 2\alpha = 2\sin\alpha\cos\alpha$

$$0 < \alpha < \frac{\pi}{2}$$

$$0 < 2\alpha < \pi$$

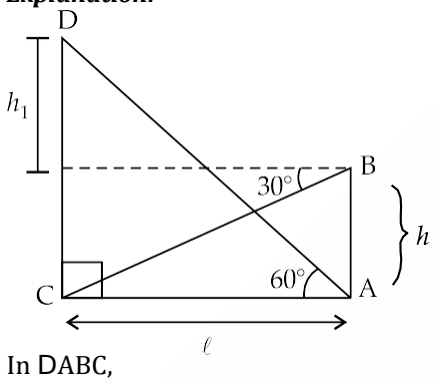
$$Q < 0, \text{ if } 0 < \alpha < \frac{\pi}{4}$$

$$Q > 0 \text{ if } \frac{\pi}{4} < \alpha < \frac{\pi}{2}$$

II is wrong

25. Option (c) is correct.

Explanation:



In $\triangle ABC$,

$$\tan 60° = \frac{l}{h}$$

$$l = \sqrt{3}h \quad \ldots (i)$$

In $\triangle ADC$,

$$\tan 60° = \frac{h_1 + h}{l}$$

$$h_1 + h = \sqrt{3}l$$

$$= \sqrt{3}(\sqrt{3}h)$$

Height of tower $= 3h$

26. Option (a) is correct.

Explanation: Given, $\frac{\sin\theta}{1-\cot\theta} + \frac{\cos\theta}{1-\tan\theta}$

$$= \frac{\sin\theta}{1-\frac{\cos\theta}{\sin\theta}} + \frac{\cos\theta}{1-\frac{\sin\theta}{\cos\theta}}$$

$$= \frac{\sin\theta}{\frac{\sin\theta - \cos\theta}{\sin\theta}} + \frac{\cos\theta}{\frac{\cos\theta - \sin\theta}{\cos\theta}}$$

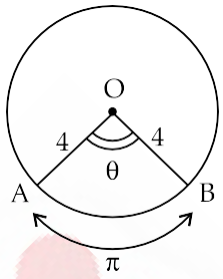
$$= \frac{\sin^2\theta}{\sin\theta - \cos\theta} - \frac{\cos^2\theta}{\sin\theta - \cos\theta}$$

$$= \frac{\sin^2\theta - \cos^2\theta}{\sin\theta - \cos\theta}$$

$= \sin\theta + \cos\theta$

27. Option (d) is correct.

Explanation:



$$\theta = \frac{\text{arc length}}{\text{radius}}$$

$$= \frac{p}{4}$$

28. Option (b) is correct.

Explanation: Given,

$$\cot^2\theta - 3\sqrt{3}\cot\theta + 6 = 0$$

$$\cot^2\theta - 2\sqrt{3}\cot\theta + 3 - \sqrt{3}\cot\theta + 3 = 0$$

$$(\cot\theta - \sqrt{3})^2 - \sqrt{3}(\cot\theta - \sqrt{3}) = 0$$

$$(\cot\theta - \sqrt{3})(\cot\theta - 2\sqrt{3}) = 0$$

$$\cot\theta = \sqrt{3}, 2\sqrt{3} \qquad \left\{\because \frac{\pi}{6} \le \theta < \frac{\pi}{2}\right\}$$

$\therefore$ $\cot\theta = \sqrt{3}$

$$\theta = \frac{p}{6}$$

$$\sin\theta + \cos 2\theta = \sin\frac{\pi}{6} + \cos\frac{\pi}{3}$$

$$= \frac{1}{2} + \frac{1}{2}$$

$$= 1$$

29. Option (d) is correct.

Explanation: I. $\sin^2\theta = \frac{(x+y)^2}{4xy}$

We know, $0 \le \sin^2\theta \le 1$

$\therefore$ $\frac{(x+y)^2}{4xy} \le 1$

$$\frac{(x+y)^2}{4xy} - 1 \le 0$$

$$\frac{(x+y)^2-4xy}{4xy} \le 0$$

$$\frac{(x-y)^2}{4xy} \le 0$$

▪ $4xy > 0$

→ $(x-y)^2 = 0$

→ $x = y$

Hence, I is wrong

Now,

II : $\sin\theta + \cos\theta = x + \frac{1}{x}$

LHS → $\sin\theta + \cos\theta$

→ $\sqrt{2}\left\{\frac{1}{\sqrt{2}}\sin\theta + \frac{1}{\sqrt{2}}\cos\theta\right\}$

→ $\sqrt{2}\sin\left(\theta + \frac{\pi}{4}\right)$

▪ $-\sqrt{2} \le$ L.H.S $\le \sqrt{2}$

R.H.S. → $x + \frac{1}{x}$

If $x > 0 \rightarrow x + \frac{1}{x} \ge 2$

If $x < 0 \rightarrow x + \frac{1}{x} \le -2$

▪ LHS ≤ RHS

Hence, II is wrong

30. Option (d) is correct.

Explanation: Given,

$m^2(\sin\theta - 1) + n^2(\sin\theta + 1) = 0$

$m^2\sin\theta - m^2 + n^2\sin\theta + n^2 = 0$

$(m^2 + n^2)\sin\theta = m^2 - n^2$

$$\sin\theta = \frac{m^2-n^2}{m^2+n^2} \quad \text{...(i)}$$

Now, $(m^2 + n^2)\cos\theta - (m^2 - n^2)\cot\theta$

$$= \cos\theta\left\{m^2+n^2-\frac{m^2-n^2}{\sin\theta}\right\}$$

$$= \frac{\cos\theta}{\sin\theta}\{(m^2+n^2)\sin\theta - (m^2-n^2)\}$$

Replace value from equation (i)

$= \cot\theta\{(m^2 + n^2) - (m^2 - n^2)\}$

$= 0$

31. Option (d) is correct.

Explanation: Given, $\sin\alpha + \cos\alpha = \sqrt{2}$

$$\frac{1}{\sqrt{2}}\sin\alpha + \frac{1}{\sqrt{2}}\cos\alpha = 1$$

$$\sin\alpha\cos\frac{p}{4} + \cos\alpha\sin\frac{p}{4} = 1$$

$$\sin\left(\alpha + \frac{\pi}{4}\right) = 1$$

$$\alpha + \frac{p}{4} = \frac{p}{2} \quad \left\{\because 0 < \alpha < \frac{\pi}{2}\right\}$$

Now, → $\alpha = \frac{p}{4}$

$$\sin^3\alpha - \cos^2\alpha = \left(\sin\frac{p}{4}\right)^3 - \left(\cos\frac{\pi}{4}\right)^3$$

$$= \left(\frac{1}{\sqrt{2}}\right)^3 - \left(\frac{1}{\sqrt{2}}\right)^3$$

$= 0$

32. Option (c) is correct.

Explanation: $(1 + \cot\alpha - \operatorname{cosec}\alpha)(1 + \tan\alpha + \sec\alpha)$

$$= \left(1+\frac{\cos\alpha}{\sin\alpha}-\frac{1}{\sin\alpha}\right)\left(1+\frac{\sin\alpha}{\cos\alpha}+\frac{1}{\cos\alpha}\right)$$

$$= \frac{(\sin\alpha+\cos\alpha-1)(\sin\alpha+\cos\alpha+1)}{\sin\alpha\cos\alpha}$$

$$= \frac{(\sin\alpha+\cos\alpha)^2-1}{\sin\alpha\cos\alpha}$$

$$= \frac{\sin^2\alpha+\cos^2\alpha+2\sin\alpha\cos\alpha-1}{\sin\alpha\cos\alpha}$$

$$= \frac{1+2\sin\alpha\cos\alpha-1}{\sin\alpha\cos\alpha} \quad (\because \sin^2\alpha + \cot^2\alpha = 1)$$

$$= \frac{2\sin\alpha\cos\alpha}{\sin\alpha\cos\alpha}$$

$= 2$

33. Option (a) is correct.

Explanation: $\tan\theta = \frac{\sin\alpha - \cos\alpha}{\sin\alpha + \cos\alpha}$

divide by cos α in numerator and denominator

$$\tan\theta = \frac{\tan\alpha - 1}{1 + \tan\alpha}$$

$$= \frac{\tan\alpha - \tan\frac{\pi}{4}}{1 + \tan\alpha\tan\frac{\pi}{4}}$$

$$\tan\theta = \tan\left(\alpha - \frac{\pi}{4}\right)$$

▪ $\theta = \alpha - \frac{\pi}{4}$

$$\sin\theta = \sin\left(\alpha - \frac{\pi}{4}\right)$$

$$\sin\theta = \sin\left(\alpha - \frac{\pi}{4}\right)$$

$$\sin\theta = \sin\alpha\cos\frac{\pi}{4} - \cos\alpha\sin\frac{\pi}{4}$$

$$\sin\theta = \frac{\sin\alpha}{\sqrt{2}} - \frac{\cos\alpha}{\sqrt{2}}$$

$$\sqrt{2}.\sin\theta = \sin\alpha - \cos\alpha$$

34. Option (a) is correct.

Explanation: Given,

$E = (\sin\alpha + 2)(\sin\alpha + 4)(\sin\alpha - 2)(\sin\alpha - 4)$

$\because -1 \le \sin\alpha \le 1$

$\therefore E \le 0$

35. Option (b) is correct.

Explanation: $\tan 3x.\tan 6x = 1$

$$\frac{\sin 3x}{\cos 3x}.\frac{\sin 6x}{\cos 6x} = 1$$

$$\sin 3x \sin 6x = \cos 6x \cos 3x$$

$$\cos 6x \cos 3x - \sin 6x \sin 3x = 0$$

$$\cos(6x + 3x) = 0$$

$\therefore$ $9x = 90°$

$x = 10°$

36. Option (a) is correct.

Explanation:

$$\frac{(a-b)^2}{(b-c)(c-a)} + \frac{(b-c)^2}{(c-a)(a-b)} + \frac{(c-a)^2}{(a-b)(b-c)} - 3$$

Let $a - b = x$

$b - c = y$

$c - a = z$

$x + y + z = 0$

$$= \frac{x^2}{yz} + \frac{y^2}{zx} + \frac{z^2}{xy} - 3$$

$$= \frac{x^3 + y^3 + z^3}{xyz} - 3$$

$(\because x + y + z = 0)$

$$= \frac{3xyz}{xyz} - 3$$

$= 3 - 3$

$= 0$

37. Option (d) is correct.

Explanation: Given,

$$\frac{100 \times 99 \times 98 \times \ldots \times 1}{100^n}$$

$$\rightarrow \frac{100!}{100^n}$$

$$\rightarrow \frac{100!}{(2\times 5)^{2n}}$$

Now, $100! = 2^{\alpha}.3^{\beta}.5^{\gamma}$

$$\gamma = \left\lfloor \frac{100}{5} \right\rfloor + \left\lfloor \frac{100}{5^2} \right\rfloor + \left\lfloor \frac{100}{5^3} \right\rfloor$$

$= 20 + 4$

{[.]} = Greatest integer function

$= 24$

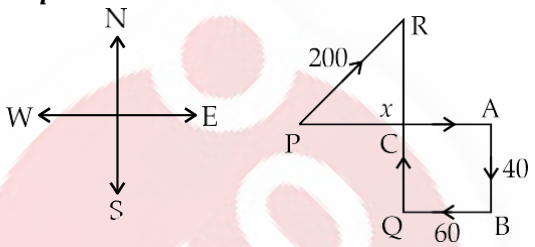
$5^{\gamma} = 5^{24}$

$(25)^n = (5^2)^{12}$

Hence, maximum value of $n = 12$

38. Option (b) is correct.

Explanation:



$PA = x$

$AB = 40$

$BQ = 60$

$QC = 40$

$QR = 200$

$PR = 200$

$PC = x - 60$

$CR = 200 - 40$

$= 160$

Now, In $\triangle CPR$

$PR^2 = PC^2 + CR^2$

$(200) = (x - 60)^2 + (160)^2$

$(200)^2 - (160)^2 = (x - 60)^2$

$(200 + 160)(200 - 160) = (x - 60)^2$

$(x - 60)^2 = 360 \times 40$

$x - 60 = 6 \times 2 \times 10$

$x = 120 + 60$

$x = 180$

39. Option (b) is correct.

Explanation: Given, x, y and $z \in I$

$x^2 + y^2 + z^2 = 3$

$\because x, y, z \in \{-1, 1\}$

$xy + yz + zx$ can be $3, -1$

$xy + yz + zx = 3$ if $x = y = z = 1$

or $x = y = z = -1$

$xy + yz + zx = -1$ either one of them -1 or two of them -1.

Let, $x = -1, y = 1, z = 1$

then, $xy + yz + zx = -1 + 1 - 1$

$= -1$

40. Option (c) is correct.

Explanation: Given, $x + y + z = 10$ ...(i)

$xy + yz + zx = 18$ ...(ii)

$(x + y + z)^2 = x^2 + y^2 + z^2 + 2(xy + yz + zx)$

$10^2 = x^2 + y^2 + z^2 + 2(18)$

$x^2 + y^2 + z^2 = 64$

Now,

$x^3 + y^3 + z^3 - 3xyz = (x + y + z)(x^2 + y^2 + z^2 - xy - yz - zx)$

$= 10(64 - 18)$

$= 460$

41. Option (a) is correct.

Explanation: $\sqrt{17 - 4\sqrt{15}} + \sqrt{8 - 2\sqrt{15}}$

$= \sqrt{12 - 4\sqrt{15} + 5} + \sqrt{5 - 2\sqrt{15} + 3}$

$= \sqrt{(2\sqrt{3} - \sqrt{5})^2} + \sqrt{(\sqrt{5}) - (\sqrt{3})^2}$

$= 2\sqrt{3} - \sqrt{5} + \sqrt{5} - \sqrt{3}$

$= \sqrt{3}$

42. Option (c) is correct.

Explanation: S= 36 + 33 + 30 +.....

$d = 33 - 36 = -3$

General term of A.P.

$t_n = a + (n - 1)d$

$o = 36 + (n - 1)(-3)$

$\frac{36}{3} = n - 1$

$n = 13$

S maximum $= \frac{13}{2}(36 + 0)$

$= 13 \times 18$

$= 234$

43. Option (c) is correct.

Explanation: According to the given condition in problem,

$m = 12\lambda + 4$

$n = 12t + 6$

$\lambda, t \in N$

I : $(m + n) = 12\lambda + 4 + 12t + 6$

$= 12(\lambda + t) + 10$

▪ if $(m + n)$ is divided by 12, then the remainder is 10

I is true.

II. $m - n = 12\lambda + 4 - 12t - 6$

$= 12(\lambda - t) - 2$

$m - n = 12(\lambda - t - 1 + 1) - 2$

$= 12(\lambda - t - 1) + 10$

II is true

44. Option (b) is correct.

Explanation: Given,

$x + y : y + z : z + x = 3 : 5 : 6$

▪ $x + y = 3t$

$y + z = 5t$

$z + x = 6t$

▪ $2(x + y + z) = 28$ ...(i)

▪ $3t + 5t + 6t = 28$

$t = 2$

▪ $x + y = 6$ ..(ii)

$y + z = 10$ ...(iii)

$z + x = 12$ ...(iv)

From equation (i), (ii), (iii) and (iv) we get,

$x = 4$

$y = 2$

$z = 8$

Now, $x^2 + y^2 + z^2 = 4^2 + 2^2 + 8^2$

$= 16 + 4 + 64$

$= 84$

45. Option (a) is correct.

Explanation: Let numbers be a, b.

$\frac{a + b}{a - b} = 5$

$a + b = 5a - 5b$

$6b = 4a$

$\frac{a}{b} = \frac{3}{2}$

Let, $\lambda = \frac{a^2 + b^2}{a^2 - b^2}$

$\lambda = \frac{\left(\frac{a}{b}\right)^2 + 1}{\left(\frac{a}{b}\right)^2 - 1}$

$= \frac{\left(\frac{3}{2}\right)^2 + 1}{\left(\frac{3}{2}\right)^2 - 1}$

$= \frac{9 + 4}{9 - 4}$

$= \frac{13}{5}$

46. Option (b) is correct.

Explanation: Let usual speed = u

usual time = t

Total distance = ut

Now according to the problem,

$\frac{ut}{(3u/5)} = t + 20$

$5t = 3t + 60$

$2t = 60$

$t = 30$ min

47. Option (a) is correct.

Explanation: p is a prime number

Apply Fermat's Little Theorem

Fermat's Theorem

If p is a prime number and N and p are coprime to each other than the remainder of $\frac{N^{p-1}}{P}$ is always 1

48. Option (a) is correct.

Explanation: Let $N = 24^3 - 16^3 - 8^3$

$= 8^3 (27 - 8 - 1)$

$= 2^9 (18)$

$N = 2^{10} \times 3^2$

∴ Number of factors = (10 + 1) (2 + 1)

$= 33$

49. Option (c) is correct.

Explanation: $S = p\left(1+\frac{r}{100}\right)^n$

$p\left(1+\frac{20}{100}\right)^n > 2p$

$\left(\frac{6}{5}\right)^n > 2$

$(1.2)^n > 2$

$(1.2)^4 = 2.07$

$\rightarrow \quad n = 4$

50. Option (d) is correct.

Explanation: Let length of train = l

Length of station = x

Length of another station = y

Speed of train = u

$$t = \frac{l+x}{u}$$

$$ut = l + x \quad \text{...(i)}$$

$$2t = \frac{l+y}{u}$$

$$2ut = l + y \quad \text{...(ii)}$$

From equations (i) and (ii),

$$y - x = ut$$

$$u = \frac{y-x}{t}$$

From equation (i),

$$y - x = l + x$$

$$l = y - 2x \quad \text{... (iii)}$$

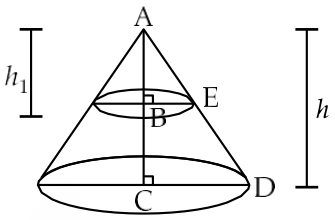
Required time $= \frac{x+y+l}{u}$

$$= \frac{x+y+y-2x}{(y-x)/t}$$

$$= \frac{(2y-x)t}{y-x}$$

51. Option (a) is correct.

Explanation:



$BE = k$

$CD = \frac{5k}{4}$

$\triangle ABE \sim \triangle ACD$

$\frac{AB}{AC} = \frac{BE}{CD}$

$\frac{h_1}{h} = \frac{4}{5}$

$\rightarrow \quad h = \frac{5h_1}{4}$

$h - h_1 = k$

$\frac{5h_1}{4} - h_1 = k$

$h_1 = 4k$

$h = 5k$

$l = \sqrt{(5k)^2 + \left(\frac{5k}{4}\right)^2}$

$l = \frac{5k}{4}\sqrt{17}$

$l_1 = \sqrt{(4k)^2 + k^2}$

$l_1 = k\sqrt{17}$

Total surface area $= \pi \times \frac{5k}{4} \times \frac{5k\sqrt{17}}{4}$

$-\pi k.k\sqrt{17} + \pi k^2 + \pi\left(\frac{5k}{4}\right)^2$

$= \pi k^2 \left|\left(\frac{9\sqrt{17}}{16} + \frac{41}{16}\right)\right|$

$\therefore 39 \pi \frac{k^2}{8}$

52. Option (a) is correct.

Explanation: Volume of frustum.

$$= \frac{1}{3}\pi\left\{CD^2h - BE^2h_1\right\}$$

$$= \frac{\pi}{3}\left\{\left(\frac{5k}{4}\right)^2 \times 5k - k^2 \times 4k\right\}$$

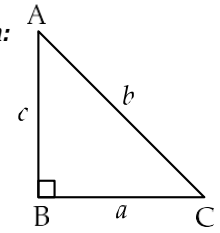
$$= \frac{\pi}{3}k^3\left\{\frac{125}{16} - 4\right\}$$

$$= \frac{\pi k^3}{48}\{125 - 64\}$$

$$= \pi k^3\left(\frac{61}{48}\right)$$

53. Option (b) is correct.

Explanation:



Given, $a + b + c = 24$ ...(i)

$a + c - b = 4$ ...(ii)

From equations (i) and (ii)

$2b = 20$

$b = 10$

$a + c = 14$ ...(iii)

$a^2 + c^2 = b^2$

$(a + c)^2 - 2ac = b^2$

$14^2 - 2ac = b^2$

$ac = 48$ ...(iv)

Area of triangle $= \frac{48}{2}$

$= 24\ cm^2$

54. Option (c) is correct.

Explanation: Perimeter = $2s = 24$

$s = 12$

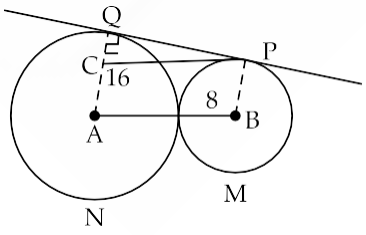
Area = 24

▪ Radius of circle $= \frac{Area}{s} = \frac{24}{12}$

$= 2$ cm

55. Option (b) is correct.

Explanation:



$QA = 16, BP = 8$

$CP = AB = 16 + 8$

$CP = 24$

$CQ = 16 - 8$

$CQ = 8$

Now, ▪ CPQ, $CP^2 = CQ^2 + PQ^2$

$(24)^2 = 8^2 + PQ^2$

$PQ^2 = 24^2 - 8^2$

$= 8^2\{9 - 1\}$

$= 64 \times 8$

$PQ = 16\sqrt{2}$

Alternative:

Formula for Common External Tangent

Length of tangent $= \sqrt{(r_1 + r_2)^2 - (r_2 - r_1)^2}$

$= \sqrt{(8 + 16)^2 - (16 - 8)^2}$

$= \sqrt{24^2 - 8^n}$

$= \sqrt{576 - 64}$

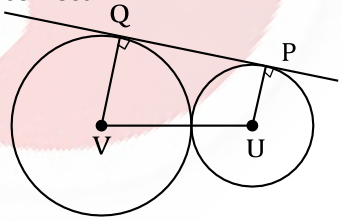
$= \sqrt{512}$

$= \sqrt{256.2}$

$= 16\sqrt{2}$

56. Option (a) is correct.

Explanation:



QV parallel to PU

Area of PQ VU $= \frac{1}{2}$ fIQ(QV + fIU)

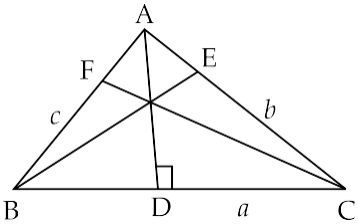
$= \frac{1}{2} \times 16\sqrt{2}(16 + 8)$

$= 8\sqrt{2} \times 24$

$= 192\sqrt{2}\ cm^2$

57. Option (b) is correct.

Explanation:



AD : BE : CF = 3 : 5 : 6

▪ $AD = 3x$

$BE = 5x$

$CF = 6x$

Area of triangle = $\Delta = \frac{BC \times AD}{2}$

$= \frac{AC \times BE}{2} = \frac{AC \times CF}{2}$

$\Delta \quad 2\Delta = a.3x = b.5x = c.6x$

$a = \frac{2\Delta}{3x}$

$b = \frac{2\Delta}{5x}$

$c = \frac{2\Delta}{6x}$

$\Delta \quad a : b : c = \frac{2\Delta}{3x} : \frac{2\Delta}{5x} : \frac{2\Delta}{6x}$

$= 10 : 6 : 5$

Hence, AB : BC : CA = 5 : 10 : 6

58. Option (d) is correct.

Explanation: $2s = 105$

$5x + 10x + 6x = 105$

$x = 5$

Δ sides of triangle are 25, 50, 30

$a = 50, b = 30, c = 25$

$2s = 105$

area of triangle = $\sqrt{s(s-a)(s-b)(s-c)}$

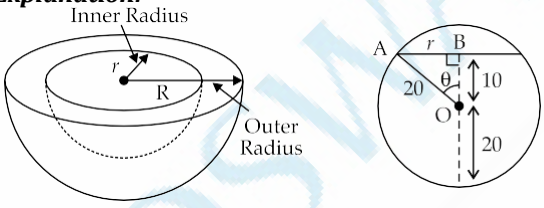
Area = $\sqrt{\frac{105}{2}\left(\frac{105}{2}-25\right)\left(\frac{105}{2}-50\right)\left(\frac{105}{2}-30\right)}$

$= \frac{1}{4}\sqrt{105 \times 55 \times 5 \times 45}$

$= 285$

59. Option (c) is correct.

Explanation:



$r^2 + 10^2 = 20^2$

$r^2 = 400 - 100$

$r^2 = 300$

$r = 10\sqrt{3}$

60. Option (a) is correct.

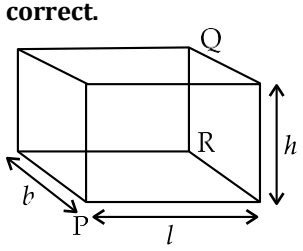
Explanation: From ΔOAB

$\cos\theta = \frac{10}{20}$

$\theta = \frac{p}{3}$

61. Option (b) is correct.

Explanation:



PQ = 14

PR = $6\sqrt{5}$

PR = $\sqrt{b^2 + l^2} = 6\sqrt{5}$

$b^2 + l^2 = 180$ ...(i)

PQ = $\sqrt{l^2 + b^2 + h^2} = 14$

$l^2 + b^2 + h^2 = 196$ ...(ii)

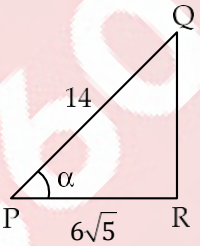
from equation (i) and (ii) we get,

$h^2 = 196 - 180$

$h = 4$

62. Option (b) is correct.

Explanation:

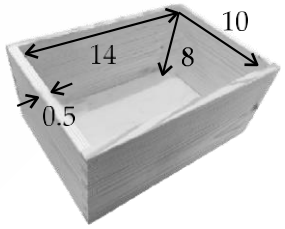


$\cos\alpha = \frac{6\sqrt{5}}{14}$

$= \frac{3\sqrt{5}}{7}$

63. Option (d) is correct.

Explanation:



Inner volume of box

$= 14 \times 8 \times 10$

$= 1120\ cm^3$

Outer length $= 14 + 2 \times 0.5$

$= 15$

Outer breadth $= 10 + 2 \times 0.5$

$= 11$

Depth $= 8 + 1 = 9$

Outer volume of box $= 15 \times 11 \times 9$

$= 99 \times 15$

$= 1485$

Volume of material = 1485 – 1120

$= 365\ cm^3$

Weight of material = $365 \times 10^{-6} \times 2000$
= 0.730 kg

64. Option (a) is correct.

Explanation:

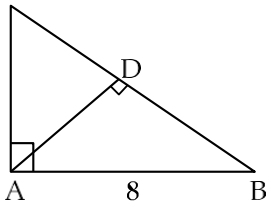
Volume of water in the box = 1120 cm^3

Weight of water = $1120 \times 10^{-6} \times 1000$
= 1.12 kg

Total weight = 1.2 + 0.73 kg
= 1.85 kg

65. Option (a) is correct.

Explanation:



$AB = 8$

$BC = 10$

In $\triangle ABC$,

$BC^2 = AB^2 + AC^2$

$10^2 = 8^2 + AC^2$

$\therefore \quad AC = 6$ cm

Area of $\triangle ABC = \frac{6 \times 8}{2} = \frac{AD \times 10}{2}$

$AD = 4.8$ cm

66. Option (b) is correct.

Explanation: $\triangle ADB \sim \triangle CAB$

$$\frac{AB}{BC} = \frac{BD}{AB}$$

$$\frac{8}{10} = \frac{BD}{8}$$

$BD = 6.4$ cm

$CD = 3.6$ cm

area of $\triangle ADC = \frac{AD \times CD}{2}$

area of $\triangle ADB = \frac{AD \times BD}{2}$

$$\frac{\text{area of } \Delta ADC}{\text{area of } \Delta ADB} = \frac{AD \times CD}{AD \times BD}$$

$$= \frac{CD}{BD} = \frac{3.6}{6.4}$$

$$= \frac{9}{16}$$

67. Option (c) is correct.

Explanation: 1 Hectare = 10^4 square metres.

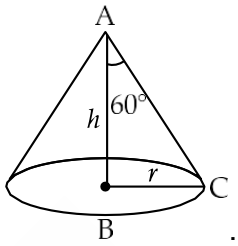
Volume of rainfall = 0.4×10^4
= 4000 cubic metre

68. Option (c) is correct.

Explanation: Weight of water (in metre tonnes)
= 4000

69. Option (c) is correct.

Explanation:



In $\triangle ABC$,

$$\frac{BC}{AC} = \sin 60°$$

$$\frac{r}{l} = \frac{\sqrt{3}}{2}$$

70. Option (a) is correct.

Explanation: $l + h + r = 9 + 3\sqrt{3}$ ...(i)

$$l = \frac{2r}{\sqrt{3}}$$

In $\triangle ABC$,

$$\tan 60° = \frac{r}{h}$$

$$h = \frac{r}{\sqrt{3}}$$

from equation (i) we get,

$$\frac{2r}{\sqrt{3}} + \frac{r}{\sqrt{3}} + r = 9 + 3\sqrt{3}$$

$$\sqrt{3}r + r = 3\sqrt{3}\left(\sqrt{3} + 1\right)$$

$$r = 3\sqrt{3}$$

$\therefore \quad h = 3$

volume $= \frac{1}{3}\pi r^2 h$

$$= \frac{1}{3}\pi(3\sqrt{3})^2 \times 3$$

volume = 27 π cubic cm.

71. Option (b) is correct.

Explanation: Let cost of article $x = a$

15 % profit on $x = a \times \frac{15}{100}$

Selling price of $x = a + \frac{15a}{100}$

$\therefore \quad a + \frac{15a}{100} = 34500$

$$a = \frac{34500}{1.15}$$

$$a = 30000$$

$$\text{Profit on } x = \frac{15}{100} \times 30000$$

$$= 4500$$

Let cost of article $y = b$

$$10\% \text{ loss} = b \times \frac{10}{100}$$

$$\therefore \quad \frac{10b}{100} = 4500$$

$$b = 45000$$

$$\text{selling price of } y = 45000 - 4500$$

$$= 40500$$

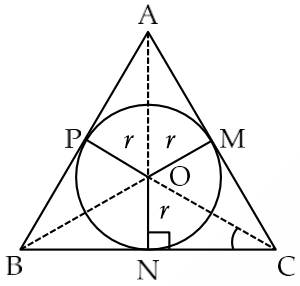
72. Option (c) is correct.

Explanation: 10 quintal = 1 metric ton

$$\therefore \quad 100 \text{ quintal} = 10 \text{ metric ton}$$

73. Option (a) is correct.

Explanation:



$$r = 2 \text{ cm}$$

$$\angle OCN = 30°$$

$$NC = \frac{a}{2}$$

In $\triangle ONC$,

$$\tan 30° = \frac{r}{a/2}$$

$$\frac{1}{\sqrt{3}} = \frac{2r}{a}$$

$$a = 2r\sqrt{3}$$

$$\therefore \quad r = 2$$

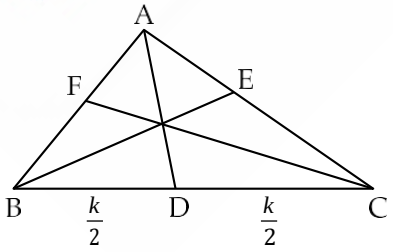
$$\therefore \quad a = 4\sqrt{3}$$

$$\text{Area of triangle} = \frac{\sqrt{3}}{4}a^2 = \frac{\sqrt{3}}{4} \times (4\sqrt{3})^2$$

$$= 12\sqrt{3} \text{ cm}^2$$

74. Option (d) is correct.

Explanation:



According to Appallonious Theorem,

$$AB^2 + AC^2 = 2AD^2 + 2BD^2$$

$$AB^2 + AC^2 = 2AD^2 + \frac{BC^2}{2}$$

$$2AD^2 = AB^2 + AC^2 - \frac{BC^2}{2}$$

$$\therefore \quad 4AD^2 = 2AB^2 + 2AC^2 - BC^2$$

$$4BE^2 = 2AB^2 + 2BC^2 - CA^2$$

$$CF^2 = 2CA^2 + 2BC^2 - AB^2$$

$$4(AD^2 + BE + CF^2) = 3(AB^2 + BC^2 + CA^2)$$

$$AD^2 + BE^2 + CF^2 = \frac{3}{4}\left\{k^2 + \left(\frac{3k}{2}\right)^2 + \left(\frac{9k}{4}\right)^2\right\}$$

$$= \frac{3k^2}{4}\left\{1 + \frac{9}{4} + \frac{81}{16}\right\}$$

$$= \frac{3k^2}{64}\{16 + 36 + 81\}$$

$$= \frac{3k^2}{64}\{133\}$$

$$= \frac{399k^2}{64}$$

75. Option (a) is correct.

Explanation: $2s = a + b + c$

Let,

$$\lambda = s(s-a)(s-b)(s-c)\left\{\frac{1}{s-a} + \frac{1}{s-b} + \frac{1}{s-c} - \frac{1}{s}\right\}$$

$$\frac{1}{s-a} + \frac{1}{s-b} + \frac{1}{s-c} - \frac{1}{s} = \frac{(s-a)+(s-b)}{(s-a)(s-b)} + \frac{s-s+c}{s(s-c)}$$

$$= \frac{2s-a-b}{(s-a)(s-b)} + \frac{c}{s(s-c)}$$

$$= c\left\{\frac{s(s-c)+(s-a)(s-b)}{s(s-a)(s-b)(s-c)}\right\}$$

$$= c\left\{\frac{s^2 - cs + s^2 - as - bs + bc}{s(s-a)(s-b)(s-c)}\right\}$$

$$= c\left\{\frac{s(2s-a-b-c)+bc}{s(s-a)(s-b)(s-c)}\right\}$$

$$= \left\{\frac{abc}{s(s-a)(s-b)(s-c)}\right\}$$

$$\{\because 2s = a + b + c\}$$

$$\lambda = \frac{s(s-a)(s-b)(s-c)abc}{s(s-a)(s-b)(s-c)}$$

$$\rightarrow \quad \lambda = abc$$

76. Option (a) is correct.

Explanation: $P = 10{,}000$

$$r = \frac{4\%}{4} = 1\%$$

$$\text{Total value} = 1000\left\{1+\frac{1}{100}\right\}^4$$

$$= 1000\left\{\frac{101}{100}\right\}^4$$

$$= 10406$$

77. Option (b) is correct.

Explanation: $p = 0.0\overline{9}$

Let, $\lambda = 70p^2 + 43p - 5$

$p = 0.0999...$

$10p = 0.999....$

∴ $9p = 0.9$

$p = 0.1$

$\lambda = 70\,(0.1)^2 + 43\,(0.1) - 5$

$= 0.7 + 4.3 - 5$

$= 0$

78. Option (b) is correct.

Explanation: Let, $N = 2^{101}$

$p = 101$

Apply Fermat's Little Theorem

Since p is a prime number and 2 is not divisible by p, we have :

- $2^{p-1} \equiv 1 \pmod p$

Multiply both sides of the congruence by 2:

- $2p \equiv 2 \pmod p$

79. Option (d) is correct.

Explanation: Roots of $x^2 + px + q = 0$ are p, q

$p + q = -p$

$pq = q$

$q \neq 0$

$\rightarrow$ $p = 1$

∴ $q = -2$

$p^2 + q^2 = 1 + 4$

$= 5$

80. Option (a) is correct.

Explanation: $x^2 + px + q = 0$ ...(i)

$x^2 + qx + p = 0$ ... (ii)

Subtract both equations,

$px + q - qx - p = 0$

$x(p - q) = p - q$

$x = 1$

Replace, $x = 1$ in equation (i),

$1 + p + q = 0$

∴ $p + q = -1$

81. Option (a) is correct.

Explanation: $x^2 - 5x + 4$ is a factor of $x^4 - px^2 + q$

Let, $f(x) = x^4 - px^2 + q$

Now, $x^2 - 5x + 4 = 0$

$x = 1, 4$

∴ $f(1) = 0 \rightarrow 1 - p + q = 0$

$f(4) = 0 \rightarrow 256 - 16p + q = 0$

∴ $256 - 16p - 1 + p = 0$

$15p = 255$

$p = 17$

∴ $q = 16$

82. Option (b) is correct.

Explanation: $px^2 + px + 4 = 0$

$x^2 + qx + q = 0$

common root is 2.

∴ $x = 2$ will satisfy both equations

$4p + 2p + 4 = 0$

$\rightarrow$ $p = \frac{-2}{3}$

$4 + 2q + q = 0$

$\rightarrow$ $q = \frac{-4}{3}$

$$p + q = \frac{-2}{3} - \frac{4}{3}$$

$= -2$

83. Option (c) is correct.

Explanation: Let $p = x^8 + x^4 + 1$

$q = x^4 + x^2 + 1$

∴ $p = x^8 + x^4 + 1 + x^4 - x^4$

$= x^8 + 2x^4 + 1 - x^4$

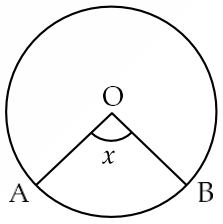
$= (x^4 + 1)^2 - (x^2)^2$

$= (x^4 + x^2 + 1)(x^4 - x^2 + 1)$

Hence, HCF of $(p, q) = x^4 + x^2 + 1$

84. Option (c) is correct.

Explanation:



radius = r

Arc length = rx

Area of sector OAB = $\frac{r^2x}{2}$

∴ $\frac{r^2x}{2} = \frac{1}{2}(rx)^2$

$x = 1$

85. Option (d) is correct.

Explanation: I LCM (p, q) is always an odd number

$\rightarrow$ wrong statement

Let $p = 2, q = 5$

LCM $(p, q) = 10$ (even)

II : Sum of their LCM and HCF is always an even number → wrong statement

Let, $p = 2, q = 3$

LCM $(p, q) = 6$

HCF $(p, q) = 1$

Sum = 6 + 1 = 7 (odd)

86. Option (a) is correct.

Explanation:

x	**frequency**
3	f
5	$f + 2$
6	$f - 3$
7	$f + 6$

$f + f + 2 + f - 3 + f + 6 = 205$

$4f = 200$

$f = 50$

87. Option (c) is correct.

Explanation:

x	f	Cumulative f
3	50	50
5	52	102
6	47	149
7	56	205

$n = 205$

∴ $\frac{n+1}{2} = \frac{206}{2}$

$= 103$

∴ median = 6

88. Option (c) is correct.

Explanation: Maximum frequency is 56.

∴ mode = 7

89. Option (b) is correct.

Explanation: Histogram is the most appropriate graphical representation.

90. Option (c) is correct.

Explanation:

x	f	xf
3	50	150
5	52	260
6	47	282
7	56	392

N = 50 + 52 + 47 + 56

∴ = 205

$(x.f)$ = 150 + 260 + 282 + 392

= 1084

mean = $\frac{1084}{205}$

= 5.29

91. Option (a) is correct.

Explanation:

Marks Obtained	No. of Students
Below 40	50
Below 50	125
Below 60	210
Below 70	315
Below 80	350

Number of students who scored between 60 and 70 marks = 315 – 210

= 105

92. Option (d) is correct.

Explanation: Below 50 marks → 125 students

Below 80 marks → 350 students

∴ More than 50 marks

= 350 – 125

= 225 students

93. Option (a) is correct.

Explanation:

Year	A	B	C	D	E
2019	20	10	15	8	20
2020	12	12	18	12	12
2021	24	14	17	14	15
2022	26	10	16	15	9
2023	22	16	14	16	11

Average circulation of newspaper D

$= \frac{8+12+14+15+16}{5}$

$= \frac{65}{5} = 13$

Circulation of newspaper D is close to its average in the year 2020 and 2021.

94. Option (d) is correct.

Explanation:

Average in year 2019 = $\frac{20+10+15+8+20}{5}$

$= \frac{73}{5}$

∴ 15

Average in year 2020 = $\frac{12+12+18+12+12}{5}$

$= \frac{66}{5}$

∴ 13

Average in year 2021 $= \frac{24+14+17+14+15}{5}$

$= \frac{84}{5}$

$\therefore 17$

Average in year 2022 $= \frac{26+10+16+15+9}{5}$

$= \frac{76}{5}$

$\therefore 15$

In year 2022 circulation of newspaper D is close to average circulation of all newspapers in that year.

95. Option (b) is correct.

Explanation:

Year	A	B	C	D	E	Average of all
2019	20	10	15	8	20	14.6
2020	12	12	18	12	12	13.2
Average of individual	20.8	12.4	16	13	13.4	

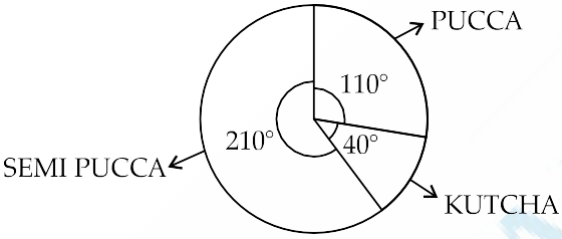
Overall average circulation of all newspapers $= \frac{14.6+13.2+16.8+15.2+15.8}{5}$

$= 15.12$

∴ Average circulation of newspapers A and C is more than 15.12 .

96. Option (c) is correct.

Explanation:

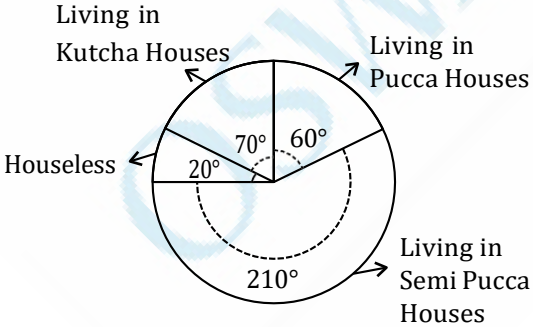


Number of PUCCA houses $= \frac{120000\times110°}{360°}$

$= 36667$

Number of Semi PUCCA houses = 70,000

Number of KUTCHA houses = 13333



Living in PUCCA houses $= \frac{210000}{360°}\times60°$

$= 350,000$

Living in KUTCHA houses $= \frac{2100000\times70°}{360°}$

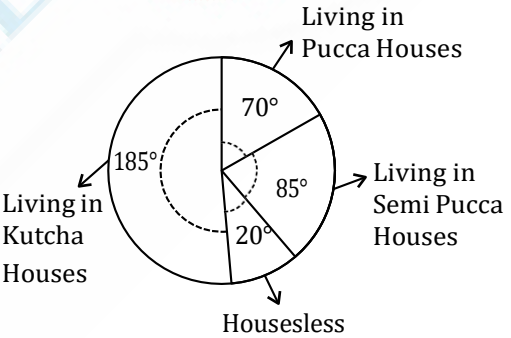
$= 408333$

Living is Semi PUCCA houses $= \frac{2100000\times210°}{360°}$

$= 1225000$

Houseless $= \frac{2100000\times20°}{360°}$

$= 116667$



Families in PUCCA Houses $= \frac{360,000\times70°}{360°}$

$= 70,000$

Families in Semi PUCCA houses = 85000

Families in Houseless = 20,000

Families in KUTCHA houses = 185000

Average of person live in every PUCCA houses $= \frac{350,000}{36667}$

$= 9.54$

97. Option (b) is correct.

Explanation: Present KUTCHA houses = 13333

New KUTCHA houses = 5000

Total KUTCHA houses = 13333 + 5000

= 18333

Angle for KUTCHA houses = $\frac{18333}{125000} \times 360°$

= 52 .79°

' 53°

Change in angle = 53° – 40°

= 13°

98. Option (b) is correct.

Explanation: Total families in KUTCHA houses

= 185000 + 300

= 185000

Number of KUTCHA houses = 13333

Average of families per houses

$= \frac{185300}{13333}$

= 13.9

99. Option (b) is correct.

Explanation:

60, 82, 63, 40, 88, 80, 24, 60, 09, 58, 44, 74
57, 60, 63, 30, 31, 64, 00, 62, 37, 54, 10, 99
35, 83, 58, 33, 92, 43, 66, 41, 78, 72, 32, 33
65, 11, 60, 17, 53, 93, 57, 03, 29, 31, 82, 50
63, 15, 41, 55, 72, 42, 32, 23, 30, 48, 67, 34

Number of students getting marks greater than or equal to 60 = 24

Number of students getting marks greater than or equal to 80 = 8

Required percentage = $\frac{24-8}{60} \times 100$

= 26.7%

100. Option (b) is correct.

Explanation: Mean of 100 observations = 50

182 taken as 82

56 taken as 6

- Corrected mean = $50 + \frac{182-82}{100} + \frac{56-6}{100}$

$= 50 + 1 + \frac{1}{2}$

= 51.5

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